# Offshoring in a Ricardian World

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# Introduction

## Technological Change

- Technological change has led to a dramatic decline in costs of communication and costs of coordinating activities performed in different locations.
- Firms in rich countries can fragment their production process and offshore an increasing share of the value chain to low-wage countries.
- Richard Baldwin (2006) refers to the phenomenon as "the second unbundling."
- This paper explores the welfare consequences of this phenomenon, thereby the impact of offshoring on rich countries.

# Introduction

## Alternative Approches

- Two approaches, both start from the notion that the unbundling of the production process entails an expansion of the set of tradable goods and services.
- First (optimist) starts from the premise that trade entails gains for all parties involved, and then concludes that fragmentation and offshoring should be good for all countries.
- Second (sceptic) reasons that increased fragmentation possibilities and lower trade costs in the limit would allow the world to reach an "integrated equilibrium" in which wages for identical workers in different countries would necessarily be equalized, i.e. wages are no longer affected by the location of workers.

- Consider a two-country model (US and RW) with labor as the only factor of production and one final good.
- Assume US has higher productivity, which entails higher wages.
- The existence of a single tradable good implies that there is no trade.
- Assume fragmentation becomes feasible, so that some labor services can now be unbundled from the production of the final good.
- If the productivity in these labor services is the same across the two countries, then trade arises, with the US specializing in the production of the final good in exchange for labor services imported from the RW via offshoring operations.
- Both countries gain from the new trade made possible by fragmentation.

# Introduction

## A Toy Model 2

- Now there are two final goods that can be traded at no cost between the US and the RW.
- US has higher productivity in good 1, productivities are the same in good 2.
- If the US is not too large relative to the world's demand for good 1, then it will specialize completely in that good and enjoy gains from trade that allow it to sustain higher wages than in the RW.
- As fragmentation becomes possible, US firms will engage in offshoring to use labor in the RW for part of their production process in good 1.
- This will enlarge US supply of good 1, which will worsen its terms of trade (TOT).
- If this process is sufficiently strong, international relative price of good 1 will converge to the US opportunity cost of this good, so that US no longer benefits from trade, and its wage level becomes equal to that in the RW.

- Fragmentation leads to new trade and to an expansion in the supply of the good in which the advanced country has a comparative advantage.
- From the point of view of the advance country, the first effect is positive and the second is negative. What is the net effect?
- One needs to capture roles played by both absolute and comparative advantage.
- The presence of an overall absolute advantage in the advanced country is a key element as it leads to the wage gap that generates incentives to offshoring.
- Comparative advantage is also necessary as it gives rise to trade in the absence of fragmentation, required for the negative TOT effect to arise.
- Eaton and Kortum (2002) Ricardian trade take both into account.
- This model allows for fragmentation and offshoring to explore their impact on wages in advanced and poor countries.

# Introduction

## Eaton Kortum (2002)

- Sector-level productivities are drawn from a distribution that is common across countries except for a technology parameter $T$.
- This technology parameter determines the location of the productivity distribution.
- Countries with a higher $T$ have better distributions.
- Countries also differ in $L$ (size of labor force, which is the only factor of production).
- Assuming away trade costs, wages are determined by the ratio of technology to size, $T/L$.
- A high $T/L$ means that the country would have many sectors in which it has absolute advantage relative to its size, leading to a high equilibrium wage.

- Note that given a fixed technology level, an increase in $L$ (immigration) would lead to a decline in $T/L$ and hence wages: this is the classic effect of size on a country's TOT in a Ricardian model.
- Fragmentation is introduced by assuming that production involves the combination of a continuum of labor services, a share of which may be offshored at no cost and with no loss of productivity.
- Fragmentation thus leads firms in high-wage countries (with a high $T/L$) to offshore a part of their production process to low-wage countries.
- This represents new trade, where high $T/L$ countries export final goods in exchange for imports of labor services through offshoring.

- Gains from new trade that takes place as well as movement toward wage equalization that harms (benefits) rich (poor) countries both present.
- 1. *Productivity effect*: captures the idea that firms experience a decline in their unit costs as they offshore prat of their production to low-wage countries.
- 2. *Term of trade effect*: due to the expansion in the supply of the good in which the rich country has comparative advantage.
- 3. *World-efficiency effect*: decline in world prices as labor is effectively reallocated from countries with low to countries with high $T/L$ ratios.

- In poor countries, only TOT and world efficiency effects present, both are positive.
- Rich countires have to deal with the negative TOT effect.
- The analysis reveals that there is always a point beyond which increased fragmentation leads to a negative effect on the real wage in the rich country.
- When fragmentation already high, further fragmentation leads to a negative TOT effect that dominates the productivity and world-efficiency effects.
- If technology gap between rich and poor countries is not too low, then the real wage in rich countries as a function of the level of fragmentation is shaped like an inverted U.
- Initially fragmentation leads to a higher real wage, but this is eventually reversed as fragmentation becomes high.
- In the limit (complete fragmentation and wage equalization), real wage in rich country necessarily lower than no fragmentation case.

- So far the discussion of fragmentation and wages takes technology levels as exogenous and can be interpreted as a short-run analysis.
- In the long-run, technology levels are endogenous, determined by research efforts and research productivity.
- Resources released by offshoring in the rich countries lead to an increased allocation of resources to research.
- This would increase the $T/L$ ratio and provide positive effects on wages not present in static analysis.
- Dynamic model with endogenous technology levels as in Eaton and Kortum (2001).

- Workers choose to work in the production sector or to do research, which leads to new ideas or technologies.
- When the technology discovered is superior to the state of the art, its owner (patent holder) earns quasi-rents that provide a return on the opportunity cost of research.
- Technology parameter $T$ can be interpreted as the "stock of ideas" in a country, and richer countries have higher stock of ideas per worker.
- Without fragmentation, the fraction of workers devoted to research is the same across countries, but countries with a higher research productivity can sustain a higher $T/L$ and hence higher wages in steady state.

- Fragmentation generates the same short-run effects, but now there is the impact on the allocation effect of workers between production and research in both rich and poor countries.
- More people in the rich country are induced to work as researchers, which in the long-run increases $T/L$ and wages, reducing the negative effect above. The opposite occurs in poor country.
- The research effect weakens the TOT effect to such an extent that it is now always dominated by the productivity effect: rich country wages increase with fragmentation in long-run.
- In steady state negative research effect compensates positive TOT effect in poor country, leaving world efficiency effect as the only source of gains.

- Increased fragmentation could have negative effects for rich countries, but these effects dissipate in time, so that the long-run effects of offshoring are always positive.
- In contrast, the long-run effects of fragmentation in poor countries are weaker than the corresponding short-run effects.
- For the rich country, the presence of opposite short- and long-run effects implies that increased fragmentation could be harmful or beneficial.
- For a special case that can be analytically solved, the paper shows that as long as the speed with which resources can be reallocated across production and research is sufficiently high, then the long-run effects dominate, and the rich country gains from offshoring.

- Implication of trade for allocation of resources to innovation (Grossman and Helpman, 1991). Other channels:
- 1. Naghavi and Ottaviano (2009): offshoring can decrease research effectiveness and growth in the North by weakening the information feedback from production to R&D. In contrast, offshoring can lead to knowledge spillovers that benefit the poor country.
- 2. Innovation can be directed at expanding the range of tasks that can be offshored. If such innovation is carried out by the poor country, benefits are higher than the present model.
- 3. Ernst (2006) and Macher and Mowery (2008): Innovation can be seen as a set of activities that are also amenable to offshoring. In the present model this type of offshoring does not take place since North does not want to offshore research to the South.

# The Model

## Basics

- Static model builds on Eaton and Kortum (2002): Ricardian trade with no transport costs.
- $N$ countries $i \in \{1, 2, ..., N\}$ and a continuum of tradable final goods indexed by $j \in [0, 1]$.
- Labor is the only factor of production, supplied inelastically at quantity $L_i$ in country $i$.
- Preferences Cobb-Douglas and symemetric: equal share of income spent on each good $j$.
- All final goods produced from a single common input: cost $c_i$ in country $i$. In Ricardian model this is labor so $c_i = w_i$. Here they differ.

# The Model

## Common Input and Fragmentation

- Common input is produced through a Leontief production function from a continuum of intermediate services indexed by $k \in [0, 1]$.
- Letting $x(k)$ represent the quantity of intermediate service $k$, output of the common input is $X = \min_k \{x(k)\}$.
- $x(k)$ is produced one to one from labor.
- If all intermediate services must be produced directly by the firm, then this collapses to the standard case $c_i = w_i$.
- Fragmentation introduced by allowing firms to costlessly offshore *at most* a certain exogenous share $\beta \in [0, 1[$ of the intermediate services. That is the "offshoring restriction".
- Focus on offshoring by country 1 (rich country) in country 2, while offshoring is not possible for all other countries.

- If $w_1 > w_2$, firms in country 1 want to exploit all opportunities for offshoring, so the unit cost of the common input is

$$c_1 = (1 - \beta) w_1 + \beta w_2. \tag{1}$$

- The common input is converted into final goods through the use of linear technologies that vary in productivity for good $j$ in country $i$.
- Letting $z_i(j)$ denote the productivity for good $j$ in country $i$, then country $i$'s unit cost of $j$ is $c_i / z_i(j)$.
- These technologies are available to all firms within a country, so the appropriate market structure is perfect competition.
- Given the absence of transport cost, price of good $j$ in all countries is simply $\min_i \{c_i / z_i(j)\}$.

- Productivities $z_i(j)$ are modelled as the realization of a random variable assumed independent across all goods and countries. In country $i$, the productivity $z_i$ for each good $j \in [0,1]$ is drawn from the Fréchet distribution

$$F_i(z) = \Pr[z_i \leq z] = \exp[-T_i z^{-\theta}], \quad (2)$$

where $T_i > 0$ and $\theta > 0$.

- Parameter $T_i$ can vary across countries and determines the location of the distribution. Higher $T_i$ implies that productivity draws are likely to be better.
- So $T_i$ is country $i$'s technology level and determines the share of goods in which it has absolute advantage relative to other countries across the continuum of goods.
- Parameter $\theta$ (common across countries) determines the variability of the draws and hence the strength of comparative advantage: A lower $\theta$ implies a stronger comparative advantage.

# The Model

## Benchmark: No Offshoring

- No offshoring means $\beta = 0$ with unit cost of the common input in country $i$ simply $w_i$ ($c_i = w_i$ for all $i$).
- The share of total income that each country spends on imports from country $i$ is equal to share of goods for which country $i$ is the least cost producer: $\pi_i = T_i w_i^{-\theta} / \Phi$, where $\Phi \equiv \sum_k T_k w_k^{-\theta}$.
- Given $w_i$, a higher $T_i$ (and given $T_i$, a lower $w_i$) implies more exports.
- Wages are determined by trade-balance conditions and in context of no trade costs are: $\pi_i Y = w_i L_i$, where $Y \equiv \sum_k w_k L_k$ is worldwide income.
- With country $N$'s labor as numeraire ($w_N = 1$), we have $w_i = \delta (T_i / L_i)^{\kappa}$, where $\delta \equiv (T_N / L_N)^{-\kappa}$ and $\kappa \equiv 1/(1+\theta)$.
- Increase in size $L_i$, holding $T_i$ constant, means a decline in country $i$'s wage. It happens through a deterioration of country $i$'s TOT and is the channel through which offshoring can lower $i$'s income level.

- When $\beta > 0$, cost of common input in country 1 differs from wage because of possibility of indirectly using labor at a cheaper cost $w_2$ in country 2.
- If $w_1 > w_2$, then the offshoring restriction is binding, and $c_1$ is given by (1).
- Since a share $1 - \beta$ of total quantity of the common input is produced domestically, full employment condition in country 1 entails $(1 - \beta)X = L_1$.
- Total labor used in country 2 via offshoring, $\beta X$, is then equal to $\alpha L_1$ where $\alpha \equiv \beta/(1 - \beta)$.
- Since other countries do not engage in offshoring, $c_i = w_i$ for all $i \neq 1$ and import share in equilibrium are now for all $i$

$$\pi_i = T_i c_i^{-\theta} / \Phi. \tag{3}$$

- Trade balance for countries 1 and 2:

$$\begin{aligned} \pi_1 Y &= w_1 L_1 + \alpha w_2 L_1, \\ \pi_2 Y &= w_2 L_2 - \alpha w_2 L_1. \end{aligned}$$

- The term $\alpha w_2 L_1$ is the value of intermediate services imported by country 1 from country 2.
- Combining (3) for 2 with its trade balance yields

$$w_2 = \delta(T_2/\tilde{L}_2)^{\kappa}, \tag{4}$$

where $\tilde{L}_2 \equiv L_2 - \alpha L_1$ is number of workers left in country 2 for production given that $\alpha L_1$ workers are devoted to offshoring services for country 1.
- Note that country 2's wage is increased by offshoring, i.e. $w_2'(\alpha) > 0$ because less workers for production increases $T_2/\tilde{L}_2$ improving its TOT.

- Combining (3) for 1 with its trade balance implies

$$(1-\beta)w_1 + \beta w_2 = \delta(T_1/\tilde{L}_1)^{\kappa}, \quad (5)$$

where $\tilde{L}_1 \equiv (1+\alpha)L_1$ is the "effective" amount of labor devoted to production in country 1, once we take into account the extra labor used through offshoring.
- Equation (5) shows two opposite effects on wages in country 1:
- 1. increase in effective number of workers in production ($\tilde{L}_1 > L_1$), which worsens TOT: *Terms of Trade Effect.*
- 2. decline in costs thanks to cheaper labor in country 2 through offshoring ($w_2 < w_1$): *Productivity Effect.*

- Equilibrium wages if resource constraint $\alpha L_1 \leq L_2$ and condition $w_1 > w_2$ implying $T_1/\tilde{L}_1 > T_2/\tilde{L}_2$ are satisfied.
- Letting $\eta \equiv (T_1/L_1)/(T_2/L_2)$ inequality can be written as

$$\eta(1 - \alpha L_1/L_2) > 1 + \alpha. \tag{6}$$

- We assume that $\eta > 1$ which means with no offshoring we have $w_1 > w_2$, then (6) is satisfied for $\alpha = 0$.
- As $\alpha$ increases LHS falls, RHS increases. The $\alpha$ that brings equality:

$$\bar{\alpha} \equiv \frac{\eta - 1}{1 + \eta L_1/L_2}.$$

- (6) is satisfied only if $\alpha < \bar{\alpha}$, country 2 resource constraint also satisfied.
- If $\alpha < \bar{\alpha}$ equilibrium characterized by (4)-(5). If $\alpha \geq \bar{\alpha}$ equilibrium entails $w_1 = w_2$, offshoring restriction is not binding, equilibrium also (4)-(5) but with $\bar{\alpha}$.
- Note $\alpha \geq \bar{\alpha}$ allows an integrated equilibrium with FPE: *Full Offshoring*.

- Since economies 1 and 2 are effectively integrated through offshoring, we can consider them as a single region $m$ with no offshoring.
- We have $L_m = L_1 + L_2$ using best technology available for each good so that $z_m(j) = \max\{z_1(j), z_2(j)\}$ implying that $z_m(j)$ is distributed Frèchet with parameters $\theta$ and $T_m \equiv T_1 + T_2$.
- Share of world income spent on imports from region $m$ is given by

$$\pi_m = T_m w_m^{-\theta} / \Phi,$$

where $\Phi \equiv T_m w_m^{-\theta} + \Phi_{-m}$ and $\Phi_{-m} \equiv \sum_{k \neq 1,2} T_k w_k^{-\theta}$.
- Total income: $w_m L_m$. Trade balance condition: $\pi_m Y = w_m L_m$.
- Just as the case of no offshoring above, we have $w_m = \delta(T_m / L_m)^{\kappa}$.

- Cosider impact on real wage $w_1/P$, with $P$ the price index of a unit of utility equal to $P = \tilde{\gamma}\Phi^{-1/\theta}$, where $\tilde{\gamma} \equiv e^{-\gamma/\theta}$, and $\gamma$ is the Euler constant.
- Since $\Phi = \sum_k T_k c_k^{-\theta}$, price index implies that higher technology levels or lower unit costs lead to lower prices.
- It follows that $P$ is lower under full offshoring than no offshoring due to higher efficiency obtained when labor effectively reallocates from country 2 to 1.
- Two opposite effects on the real wage in country 1 as we move from no offshoring to full offshoring: 1. TOT effect which decreases relative wage $w_1$, 2. *world-efficiency effect*, which lower price index $P$.
- No productivity effect (no wage gap) so country 1 does not gain from trading services with country 2. Cost saving from cheaper labor of country 2 are dissipated as more offshoring is undertaken by country 1 firms (no gains from trade when international price = autarky price).

# The Model

## Proposition 1

- TOT effect always dominates the world efficiency effect so that $w_1 / P$ is always lower under full offshoring. Recalling that $w_2$ increases with offshoring:
- *There is full offshoring if $\alpha \geq \bar{\alpha}$, where $w_1$ and $w_1 / P$ are lower, and $w_2$ and $w_2 / P$ are higher than than no offshoring.*
- To see why TOT effect dominates, think of full offshoring equilibrium equivalent to $\Delta$ workers reallocated from country 2 to 1, where $\Delta$ given implicitly by $T_1 / (L_1 + \Delta) = T_m / L_m = T_2 / (L_2 - \Delta)$. This also gives $w_1 = w_2 = w_m$.
- Reallocation of workers two steps: increase in $L_1$ by $\Delta$ and decrease in $L_2$ by $\Delta$. Ricardian theory implies that both lead to decline in country 1 real wage $w_1 / P$.
- Allow for costs for trade in final goods between the two countries and RW. Equilibrium wage of 1 and 2 no longer $w_m = \delta(T_m / L_m)^\kappa$, but still increasing in $T_m / L_m$. Since $T_m / L_m < T_1 / L_1$, $w_m$ lower than $w_1$ under no offshoring. Also real wage lower.

- Solving for $w_1$ from (5) yields

$$w_1 = (1+\alpha)\tilde{w}_1 - \alpha w_2,$$

where $\tilde{w}_1 \equiv \delta(T_1/\tilde{L}_1)^\kappa$ is country 1 wage with no offshoring if its labor supply was $\tilde{L}_1$: if offshoring only generated TOT but no productivity effect.

- Differentiating with respect to $\alpha$ and simplifying yields

$$w_1' = (1+\alpha)\tilde{w}_1' - \alpha w_2' + (w_1 - w_2)/(1+\alpha).$$

- First term on RHS: TOT effect. It is negative because $\tilde{w}_1' = -\kappa\tilde{w}_1/(1+\alpha) < 0$. Intuitively, as $\alpha$ increases, "effective" supply $\tilde{L}_1$ increases leading to decline in wage by worsening country 1's TOT.
- Second term: negative because $w_2$ is increasing in $\alpha$. This is a demand effect: as offshoring increases, this pushes up country 2's wages, and this hurts country 1, which uses country 2's labor as an input.
- Third term: productivity effect, positive as long as $w_1 > w_2$. Access to cheaper labor, country 1 achieves decline in its costs, leading to higher wages.

# The Model

- Two points to characterize the net marginal effect of offshoring $w_1'(\alpha)$:
- 1. Productivity effect depends positively on wage difference $w_1 - w_2$, which in turn is increasing the ratio of per capital technology levels in country 1 relative to 2, or $\eta$. Thus, $w_1'(\alpha)$ more likely to be positive if $\eta$ large:

$$w_1'(0) = w_2'(0)[(1-\kappa)\eta^{\kappa} - 1];$$

thus, $w_1'(0) \gtrless 0$ according to whether $\eta \gtrless (1-\kappa)^{-1/\kappa}$.

- 2. As $\alpha$ gets close to $\bar{\alpha}$, wage difference $w_1 - w_2$ goes to zero and the productivity effect vanishes, so $w_1'(\alpha)$ is negative for $\alpha$ close enough to $\bar{\alpha}$.
- *Consider $\alpha$ in interval $[0, \bar{\alpha}]$. Function $w_1(\alpha)$ is concave. For $\eta \leq (1-\kappa)^{-1/\kappa}$, the curve $w_1(\alpha)$ is always decreasing, whereas for $\eta > (1-\kappa)^{-1/\kappa}$ it is shaped like an inverted U.*

- For real wages we need to bring in world efficiency effect. Offshoring decreases price index $P$. Intuitively, an increase in $\alpha$ implies more possibilities to trade and this increases worldwide efficiency.
- *Price index $P$ is decreasing in $\alpha \in [0, \bar{\alpha}]$.*
- Since $w_2(\alpha)$ is increasing, then clearly $w_2(\alpha)/P(\alpha)$ is also increasing. Similarly if $w_1(\alpha)$ is increasing, then $w_1(\alpha)/P(\alpha)$ will also be increasing. What happens if $w_1(\alpha)$ is decreasing?
- *Consider $\alpha$ in interval $[0, \bar{\alpha}]$. There exists $\hat{\eta}$, such that if $\eta \leq \hat{\eta}$ then $w_1/P$* decreasing in $\alpha$, *while if $\eta > \hat{\eta}$, then $w_1/P$ is an inverted U as function of $\alpha$.*
- When fragmentation sufficiently high, further increase hurts rich country because productivity and world-efficiency effects dominated by TOT and demand effects.

- Up to now technology levels were fixed, here we endogenize them in a fully dynamic model, where the "short run" analysis is equivalent to the static one above.
- Technological progress is modelled as people choosing to do research or work in the productive sector.
- $L_{it}$ still denotes workers engaged in production, total labor force now is $L_{it}^F$, and $R_{it}$ is the number of people working as researchers in country $i$ at time $t$ so that full employment condition is $R_{it} + L_{it} = L_{it}^F$.
- Assume that $L_{it}^F$ grows at a constant rate $g_L$ common across countries.
- Assume reallocation of workers between production and research to be sluggish: $L_{it}$ is a state variable and fixed in the short run as before.
- Assume people are born as producers or reserachers in proportion to current population, and at each $t$ get a chance to switch sectors at a constant/exogenous probability $\upsilon_P(\upsilon_R)$ for those in production (research).

# The Dynamic Model

## Ideas 1

- A researcher in country $i$ draws technologies or "ideas" at a Poisson rate $\phi_i$. This reflects research productivity and may vary across countries.
- Let $T_{it}$ be the total number of ideas that have been generated in country $i$ up to time $t$, then $\dot{T}_{it} = \phi_i R_{it}$ and

$$T_{it} = \phi_i \int_0^t R_{is} ds. \tag{7}$$

- Each idea has two characteristics: the good $j \in [0, 1]$ to which it applies, and its productivity $q$, each modelled as the realization of a random variable: $j$ is distributed uniformly over interval $[0, 1]$, while $q$ is distributed Pareto with parameter $\theta > 1$.
- Formally, for $q \geq 1$,it is assumed that

$$H(q) = \Pr[q' \leq q] = 1 - q^{-\theta}.$$

# The Dynamic Model

## Ideas 2

- Let $z_{it}(j)$ be the maximum $q$ over ideas that apply to good $j$ in country $i$ and time $t$. It can be shown that distribution of $z_{it}(j)$ has the Fréchet form as in (2) with $T_{it}$ given by (7).
- This means that the process for the arrival of ideas here leads to the Fréchet productivity frontier postulated in the static model, with $\theta$ in the Fréchet distribution coming from parameter $\theta$ in the Pareto distribution of the quality of ideas and $T_i$ growing over time and being equal to the stock of ideas in country $i$ at time $t$.
- Researchers sell their ideas to firms that engage in Bertrand competition with other firms in the worldwide market for consumer goods.
- Considering competition for a particular good, only firms holding best ideas for this good within some country have a chance of surviving competition in the international market.
- Thus, country that captures worldwide market for good $j$ at time $t$ is given by $\arg\min \{c_i / z_{it}(j)\}$.

# The Dynamic Model

## Firms and Profits

- The firm that captures worldwide market for a good will make positive quasi-profits by charging a mark-up that depends on the second-least unit cost.
- In Eaton and Kortum (2001) this mark-up is also distributed Pareto with $\theta$ or $m \sim H(m)$.
- This is the distribution for the mark-up charged by firms from any country, and is constant.
- Let $Y_t$ denote worldwide income at $t$, which is also worldwide expenditure on every good.
- Hence if a firm charges mark-up $m$, then its profits are $Y_t(1 - (1/m))$, and total worldwide profits are

$$Y_t \int_1^\infty (1 - (1/m)) dH(m) = \kappa Y_t.$$

- Since country $i$ captures the worldwide market for a share $\pi_{it} = T_{it} c_{it}^{-\theta} / \Phi_t$ of goods, its total income is $\pi_{it} Y_t$ and its total profits are a share $\kappa$ of that.

# The Dynamic Model

## Utility

- Letting $d_{it}$ be the probability of a random idea from country $i$ having a market at time $t$, then expected profits of a random idea from country $i$ are $\kappa d_{it} Y_t$.
- Thus expected discounted value of a random idea from country $i$ at $t$ is

$$V_{it} = \kappa \int_t^{\infty} e^{-\rho(s-t)} (P_t / P_s) d_{is} Y_s ds,$$

where $\rho$ is the discount rate in consumers' intertemporal utility function, $u_t = \int_t^{\infty} e^{-\rho(s-t)} U_s ds$ and $P_t$ is the price index.
- Eaton and Kortum (2001) show that $d_{it} = \pi_{it} / T_{it}$: recall $\pi_{it}$ is share of worldwide spending devoted to purchases from country $i$ and also the probability that $i$ is the least-cost producer for a good.
- For an idea in $i$ to have a market it must be the best idea in $i$, *and* it must beat the competition from all other countries.
- Probability that a random idea is the best one in $i$ is simply $1 / T_{it}$, whereas the probaiblity that the idea beats the foreign competition is $\pi_{it}$.

# The Dynamic Model

## Short-Run Analysis

- As $L_{it}$ and $T_{it}$ are fixed, the only difference between the static and the dynamic model regarding the short-run implications of offshoring is the market structure.
- In static model there is perfect competition, whereas in the dynamic model technologies are owned by firms that engage in Bertrand competition.
- Existence of mark-ups and profits under Bertrand do not change comparative statics (profit share is common across countries) because:
- trade balance now requires exports of goods and offshoring services plus domestic sales be equal to wages plus imports of offshoring services *plus profits*.
- Since value of exports and domestic sales of goods is $\pi_{it} Y_t$, and profits are a share $\kappa$ of this value, we can say that trade balance requires $(1-\kappa)\pi_{it} Y_t$ plus exports of offshoring services to equal wages paid to domestic and foreign workers (through offshoring).

# The Dynamic Model

## Steady State Analysis 1

- In SS, $r_{it} \equiv R_{it}/L_{it}^F$ will be constant, so the growth rate of stock of ideas $T_{it}$ is $\dot{T}_{it}/T_{it} = g_L$, and its level would be
$$T_{it} = (\phi_i r_i / g_L) L_{it}^F. \tag{8}$$
- Choice of country $N$'s labor as numeraire implies SS wages to be constant, $w_{it} = w_i$, so price index $(P = \tilde{\gamma}\Phi^{-1/\theta})$ falls at rate equal to $\theta g_L$, so $P_s = P_t e^{-(g_L/\theta)(s-t)}$.
- In SS $\pi_{it}$ is also constant. Also, equality between sales and expenditure (trade balance) entails $\pi_i Y_s = Y_{is}$, implying
$$V_{it} = \kappa \int_t^\infty e^{-(\rho - g_L/\theta)(s-t)} (Y_{is}/T_{is}) ds. \tag{9}$$
- Incomes in countries 1 and 2 respectively are:
$$Y_{1t} = w_1 L_{1t} + w_2 \alpha L_{1t} + \kappa Y_{1t},$$
$$Y_{2t} = w_2 L_{2t} + w_2 \alpha (1 - r_1) \varphi L_{2t}^F + \kappa Y_{2t},$$

where $\varphi \equiv L_{1t}^F / L_{2t}^F$.

- Using $L_{1t} = (1 - r_1) L_{1t}^F$, and solving for $Y_{1t}$, plugging the resulting expression for $Y_{1t}$ into (9), using (8), and assuming $\theta\rho > g_L$ yields

$$V_1 = w_1 \left[ 1 - r_1 + \alpha(1 - r_1) \frac{w_2}{w_1} \right] \left( \frac{g_L}{\phi_1 r_1} \right) \frac{1}{\theta\rho - g_L},$$

$$V_2 = w_2 \left[ 1 - r_2 + \alpha(1 - r_1) \varphi \right] \left( \frac{g_L}{\phi_2 r_2} \right) \frac{1}{\theta\rho - g_L}.$$

- For all other countries, the corresponding expected value of an idea can be derived from the previous results by simply plugging $\alpha = 0$:

$$V_i = w_i (1 - r_i) \left( \frac{g_L}{\phi_i r_i} \right) \frac{1}{\theta\rho - g_L}.$$

- In equilibrium, expected payoff to research must be equal to wage in every country. This entails $\phi_i V_i = w_i$. For countries $i \neq 1, 2$ this can be solved to yield
$$r_i = r \equiv g_L / \theta \rho. \tag{10}$$
- This implies that differences in $\phi_i$ do not affect proportion of workers engaged in research.
- For countries 1 and 2 the equilibrium conditions (after some simplification) are
$$r_1 / r = 1 + \alpha(1 - r_1) w_2 / w_1, \tag{11}$$
$$r_2 / r = 1 - \alpha(1 - r_1) \varphi. \tag{12}$$
- Given the wage ratio $w_2 / w_1$, these two equations determine the research intensities in countries 1 and 2.

- Using (8) and $L_{it} = (1 - r_i)L_{it}^F$ yields

$$\frac{T_{is}}{L_{is}} = \frac{\phi_i r_i}{g_L(1 - r_i)}. \tag{13}$$

- Thus from (10) and $w_i = \delta(T_i/L_i)^\kappa$ we see that for $i \neq 1, 2$ SS wage is $w_i = (\phi_i/\phi_N)^\kappa$: wages hence only differ because of differences in research productivity $\phi_i$.
- With no offshoring ($\alpha = 0$) wages in countries 1 and 2 are also given by this expression. Thus, condition that $w_1 > w_2$ in SS with no offshoring is that $\phi_1/\phi_2$ (this is the long-run version of condition $\eta > 1$ in static model).
- As long as resource constraint $\alpha(1 - r_1)L_{1t}^F \leq L_{2t}^F$ is satisfied, SS wages in countries 1 and 2 with $\alpha > 0$ are determined by (4)-(5) together with $L_{it} = (1 - r_i)L_{it}^F$ and equations (11)-(12).

- From (4) we get

$$w_2 = \left( \frac{\phi_2 r_2}{\phi_N r/(1-r)} \frac{1}{1 - r_2 - \alpha(1-r_1)\varphi} \right)^{\kappa}$$

- Using (12) this becomes $w_2 = (\phi_2/\phi_N)^{\kappa}$ as the case of no offshoring because the decline in $\tilde{L}_2$ generated by increased offshoring in the static model is now exactly compensated by a decline in $T_2$ caused by a decline in $r_2$.
- For $w_1$, recall from (5) that $(1-\beta)w_1 + \beta w_2 = (T_{Ns}/L_{Ns})^{-\kappa}(T_{1s}/\tilde{L}_{1s})^{\kappa}$. With endogenous research, the ratio $T_{1s}/\tilde{L}_{1s}$ now also depends on research effort as well as the extent of offshoring. From (11), (13),and $\tilde{L}_1 \equiv (1+\alpha)L_1$ we get

$$T_{1s}/\tilde{L}_{1s} = \left( \frac{T_{Ns}/L_{Ns}}{\phi_N} \right) \left( \frac{\phi_1}{w_1} \right) ((1-\beta)w_1 + \beta w_2).$$

- Equilbrium SS wage in country 1 is determined by

$$(1-\beta)w_1 + \beta w_2 = \left(\frac{\phi_1/\phi_N}{w_1}\right)^{\kappa} ((1-\beta)w_1 + \beta w_2)^{\kappa}. \qquad (14)$$

- LHS is the unit cost of the common input. RHS is proportional to $(T_{1s}/\tilde{L}_{1s})^{\kappa}$ and captures the impact of offshoring and research on country 1's TOT.
- Given assumption $\phi_1 > \phi_2$, the level of $w_1$ determined by (14) is higher than $w_2$.
- But this implies offshoring lowers unit cost of common input (i.e. LHS increasing in $\beta$). This represents the productivity effect.
- Note that an increase in $\beta$ decreases the RHS, a reflection of the negative TOT effect.
- Which effect dominates? Since $\kappa < 1$, the productivity effect always dominates, so $w_1$ is increasing in $\beta$.

# The Dynamic Model

## Steady State Analysis 6

- We have so far ignored the resource constraint in country 2 that the amount of labor used for exporting services to country 1 must be lower than its total labor force: $\alpha(1 - r_1)L^F_{1t} \leq L^F_{2t}$. It can be shown from results above that if $r > \phi_1/((\phi_1 + \phi_2)/\varphi)$ then resource constraint is satisfied for all $\alpha$.
- Otherwise there exists a level $\hat{\alpha}$ such that resource constraint is binding for $\alpha > \hat{\alpha}$, in which case the equilibrium entails wage equalization for all workers in country 2 employed in offshoring operations for country 1.
- Previous results relate to wages in countries 1 and 2 relative to a third country $N$. But it can be shown that the price index $P$ will decline with offshoring, as the efficiency gains in the static model are only expanded in this dynamic model as offshoring allows a reallocation of labor towards the activity in which they have comparative advantage (research in country 1 and production in country 2.

# The Dynamic Model

## Proposition 5

- *As long as the resource constraint in country 2 is nonbinding, an increase in $\alpha$ increases the wage in country 1, whereas the wage in country 2 is not affected. The real wages $w_i/P$ increase in all countries.*
- What happens to $r_1$ and $r_2$ as $\alpha$ increases? Equation (11) implies

$$r_1 L_{1t}^F = r[L_{1t}^F + \alpha(1 - r_1)L_{1t}^F w_2/w_1].$$

- The term $\alpha(1 - r_1)L_{1t}^F w_2/w_1$ is the number of worker in directly hired by country 1 from 2 through offshoring, adjusting for the wage ratio.
- Thus the number of people doing research in country 1 is proportion $r$ of total labor force in 1 including workers indirectly working in country 1 through offshoring. $r_1$ is thus necessarily higher with offshoring.
- Moreover, $\alpha(1 - r_1)L_{1t}^F w_2/w_1$ is increasing in $\alpha$, so it is also the case that offshoring increases country 1's research intensity $r_1$.

- Turning to country 2, rearranging equation (12) gives

$$r_2 L^F_{2t} = r(L^F_{2t} - \alpha(1 - r_1)L^F_{1t}).$$

- Analogous to result for country 1, this expression says that the number of people doing research in country 2 is a proportion $r$ of its total labor force, excluding workers producing services for export through offshoring operations.
- This implies that $r_2 < r$ as long as $\alpha > 0$. More generally, it can be shown that $r_2$ is decreasing in $\alpha$. Formally,
- *The research intensity $r_1$ in country 1 increases while the research intensity* $r_2$ in country 2 decreases as $\alpha$ increases.

# The Dynamic Model

## Transition Dynamics 1

- Imagine an unexpected increase in fragmentation at time $t_0$.
- We know that if increase in $\alpha$ is large enough, it would lead to decline in real wage in country 1 at $t_0$.
- As time goes by, however, workers in country 1 would switch from production to research, increasing $T_{1t}/L_{1t}$ and improving country 1's TOT.
- In the new SS, real wage in country 1 is higher.
- There are two opposite effects of a large (unexpected) increase in fragmentation: a negative short-run effect and a positive long-run effect.
- What is the net effect for utility at time $t = 0$?
- Assume a shock to $\alpha$ and that countries 1 and 2 are very small so that rest of the world is not affected and $P_t$ continues to fall at rate $g_L/\theta$ even after the shock.

# The Dynamic Model

## Transition Dynamics 2

- Recall wages $w_1$ and $w_2$ are determined by ratios $T_{1t}/\tilde{L}_{1t}$ and $T_{2t}/\tilde{L}_{2t}$, where $\tilde{L}_{1t} = (1+\alpha)(1-r_{1t})L_{1t}^F$ and $\tilde{L}_{2t} = (1-r_{2t})L_{2t}^F - \alpha(1-r_{1t})L_{1t}^F$.
- One can think of these ratios as a function of $\alpha$ together with ratios $x_{1t} \equiv T_{1t}/L_{1t}^F$ and $x_{2t} \equiv T_{2t}/L_{2t}^F$, and the research shares $r_{1t}$ and $r_{2t}$.
- Positive $\alpha$ shock throws system out of SS, with a low initial value of $x_{1t}$ and high initial value of $x_{2t}$ (relative to their new SS levels).
- Assuming that the exit rates $\upsilon_P$ and $\upsilon_R$ are sufficiently large and that $\upsilon_R$ is large relative to $\upsilon_P$, the equilibrium adjustment has three stages.
- Stage 1: $\phi_1 V_{1t} > w_{1t}$ and $\phi_2 V_{2t} < w_{2t}$ so there is maximal entry into research in country 1 and maximal exit from research in country 2. This stage ends when $w_{2t}$ reaches steady state $w_2$.

- Stage 2: $\phi_1 V_{1t} > w_{1t}$ and $\phi_2 V_{2t} < w_{2t} = w_2$ so maximal entry into research continues in country 1 while the constraint on exit from research in country 2 is no longer binding. This stage ends when $w_{1t}$ reaches steady state $w_1$.
- Stage 3: $\phi_i V_{it} < w_{it} = w_i$ for $i = 1, 2$, so that wages in countries 1 and 2 are at their SS values, and $r_{1t}$ and $r_{2t}$ adjust in response to the continued movement of $x_{1t}$ and $x_{2t}$, towards their SS values.
- It is clear that if $\upsilon_P$ and $\upsilon_R$ are very high, then the first two stages of the adjustment process will be short, and the adjustment will entail wages being at their new SS values most of the time.
- Since an increase in $\alpha$ increases the SS wage of country 1, this country must benefit from such a shock even if it experiences some losses in the short run.
- Country 2 also experiences a positive welfare effect, because wages are momentarily higher there after the shock, although they rapidly converge to the same level as before th eshock.